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General equilibrium under rationality hypothesis

The Walrasian equilibrium existence theorem is slightly restated with the assumptions of complete or transitive preferences.

1. INTRODUCTION

Mathematical economics is a very young science that has made much progress in these recent years and encompassing especially areas like the process of price formation, the theory of exchange, the theory of production, etc.,. Even this opinion is not going to be shared by all economists, at the present time, it can certainly be claimed, without overestimating the role played by the underlying mathematics, that this new method has already become an indispensable tool for formulating various areas of economic phenomena. It is not the purpose of this paper to discuss such questions more deeply. We only wish to point out the circumstances, which have been the principal stimulus to it.

In mathematical economics, the concept of a general equilibrium theory based on balance of supply and demand has played a central role. Roughly speaking, the situation can be described in its simplest formulation by Walras, in the following terms : In a free market, the price of each commodity bundle x^i depends on the extent to which it is demanded by consumers i. We assume here that there are m consumers. More precisely, this implies that, if at a given set of prices, the demand for a good exceeds the available supply, then its price rises, thus causing the demand to decrease, and vice-versa. It therefore appears that prices will eventually regulate themselves to values at which supply and demand exactly balance. By this mechanism it is supposed that economic equilibrium exists.

Needless to say, rigorous are investigations that have been made in favor of the conditions under which such a balance is possible. And notable among these, are the papers of Wald, especially(1), and most recently some interesting results of Arrow and Debreu(2). Arrow and Debreu have developed a distinctly original, and very interesting, proof of the existence of a competitive equilibrium, using technical methods of analysis under special assumptions. We are going here to study theorems closely resembling that of Arrow's and Debreu's 1954 theorem; and to use the rationality hypothesis developed in economic theory to investigate some properties of the concept of general equilibrium theory. The aim of this article is thus described in its title: To demonstrate that for the general equilibrium Walrasian model to be well defined and consistent(3), the hypothesis of individual rationality is needed. We refer to McKenzie (1981, also 1959), for forceful arguments in favor of introducing strong rationality on decision-makers' preferences.

For the case where instrumental rationality is not defined as the choice of actions which best satisfy an agent's objectives, Leroux (1993) showed how it was possible to reintroduce the concept of "imperfect discriminating power" of the consumers's preferences and establishes the existence of "spots of equilibria(4)" in exchange economies.

McKenzie's classical theorem is characterized above all by its use of assumptions of finiteness and convexity. That is, the model comprises a finite number of economic agents or consumers who trade in a single market under conditions of certainty. The goods are finite in number and, as a consequence, the horizon is also finite. Goods are divisible, and production is modeled either as a set of linear activities in the space of goods or as convex input-output sets belonging to a finite list of firms. Consumption sets and preference relations are also convex in an appropiate sense. Consumption and production activities are mutually independent.

The problem at hand seems to require an existence proof of a novel type. However, in recent years, this fact itself exploited by McKenzie has been improved in basic ways by Andreu Mas-Colell(5) and many other writers. A point to which we shall return later in section 3. In Section 2 we present the notation used in the classical theorem, and state the existence result established by McKenzie. The remainder of the paper is devoted to an extensive and relatively non - technical discussion of the theorems along with a number of comments. This will become clear in the text.

For the sake of clarity and conciseness the analysis is limited to pure exchange economies even generalizing the McKenzie assumptions. Thus there is no difficulty in extending the results to, for instance, the private ownership economies of Debreu's *Theory of value* (1959).

2. PRELIMINARIES AND DISCUSSION OF THE EXISTENCE THEOREM

We shall, for the most part, employ the notation used by McKenzie's 1959 theorem; with a little modification that made the Weak Axiom assumption of Revealed Preference. This axiom virtually reduces the set of consumers to one person, since it is equivalent to consistent choices under budget constraints. More precisely, we assume that, in a such context, the existence of the equilibrium becomes a simple maximum problem and advanced methods are not needed. Note that when a continuum of agents with independent preference orders are present, it has been shown by Uzawa(6) (1962) that fixed point methods are necessary.

In concrete terms, the theorem to be considered in this section will involve assumptions on the consumption sets X_i, on the total production set Y, and on the relations between these sets. Our hypotheses on the consumption sets, which lie in R^n, the cartesian product of *n* real lines, are:

ASSUMPTION I. The sets X_i are closed, bounded from below and convex and contain the null vector 0.

This assumption calls for some comment. X_i is interpreted as the set of feasible trades of the *i*th consumer. There are *m* consumers. The condition of boundedness is natural in view of the fixed time interval, and the closedness represents an idealization of a topological character which is not very restrictive when boundedness and convexity are assumed. That X_i is bounded from below means thus that there is e_i such that $x > e_i$ holds for all $x \in X_i$. Together with the fact that the null vector 0 belongs to each X_i, it means that if *x* and *x'* are two bundles in X_i then so also all bundles of the form *kx* + *k'x'* where $k, k' \geq 0$ and $k + k' \leq 1$.

The problem is now to determine how an abstract model of an economy that has been complicated in many ways for the existence theorems, by weakening principally the crucial finiteness and convexity assumptions, should operate. We must answer the following questions: (*i*) What goods shall the firms produce? (*ii*) What types of services shall the consumers supply? (*iii*) How shall the goods produced be disturbed among them? (*vi*) Will they have sufficient wealth to participate in a market economy? The result here answers these questions by exploring ways in which the standard theory of rationality is relevant to the consumer's behavior. Of course, the approach we give is going to includ all the four problems in a single question, but will not lack intuitive appeal in a context where preference may be complete and transitive. This does not appear to have been used before. In other words, what bundle x^i shall be assigned to the choice of the consumer from the set X_i? Clearly such an assignment of bundles x^i to consumers must satisfy the condition that an agent chooses or acts rationally if his preferences are rational, and he never prefers an available option to the option chosen. Obviously, the set of bundles x^i must satisfy the condition $\sum x^i \geq 0$. Thus, we are led to make the following

DEFINITION. A feasible trade of an economy is a set of bundles $(x^i)_{i = 1,2,\ldots,m}$, where $x^i \in X_i$ and $\sum x^i \geq 0$.

As the reader has probably already noted, we are not going in what follows to show how a feasible trade of an economy can be brought about by the mechanism of prices and free competition, but assume that we have a free competitive model of an economy in which each agent may choose a commodity bundle x^i which, intuitively speaking, maximizes his satisfaction. This assumption can be made without loss of generality in the sense that the assumptions on the preference preordering of consumer $i \in \{1,2,\ldots,m\}$ and the consumption sets X_i sufficient to guarantee the existence of a utility function U_i satisfying the convention adopted here(7). Thus, the choice of this consumer will clearly be a function U_i of the prices *p*. The bundle x^i choosen therefore by agents must be such that the income received from goods supplied is sufficient to pay for the goods consumed(8).

Following McKenzie, we then assume the following.

ASSUMPTION II. The sets X_i are completely ordered by a convex and closed preference relation ϕ.

Convexity of the preference relation ϕ means that $x \phi x'$ implies $x'' \phi x'$ where $x'' = \lambda x + (1 - \lambda)x'$, for $0 < \lambda < 1$. Closure of ϕ means that $x^k \rightarrow x$ and $x^{k'} \rightarrow x'$, implies $x \phi x'$.

To justify the conditions of this assumption, we are going to show here that they can be derived from simple assumptions concerning the preferences of the consumers. We assume that the commodity space $X = \sum_1^m X_i$, where *m* is the number of consumers, is a compact convex subset of R^n, and assume again that all the bundles of X are ordered by a simple ordering relation ϕ. Since ϕ is a simple ordering it satisfies :

(*i*) $x \phi x'$ and $x' \phi x''$ implies $x \phi x''$.

(*ii*) For any $x, x' \in X$, either $x \phi x'$ or $x' \phi x$.

DEFINITIONS. The ordering ϕ is called *continuous*(9) in the usual sense if $x \phi x'$ implies there exist neighborhoods V_1 of *x* and V_2 of *x'* such that $x_1 \phi x_2$ for all $x_1 \in V_1$ and all $x_2 \in V_2$.

The ordering is called *convex* (*strictly convex*) if $x \sim x'$ and $0 < \mu < 1$ implies $\mu x + (1 - \mu)x' \phi x$.

Let $\Omega_1 = \{x_1 : x_1 \phi x\}$, $\Omega_2 = \{x_1 : x \phi x_1\}$. The following property follows at once from the definitions.

LEMMA 1. The ordering ϕ is *continuous if and only if* $\Omega_1(x)$ and $\Omega_2(x)$ are *closed* for all $x \in X$.

Not quite so obvious is the following.

LEMMA 2. A *continuous* ordering *f* is *convex if and only if* $W_2(x)$ is convex for all *x* Î X.

PROOF. The suffiency of the condition is obvious. Conversely, suppose $y_1, y_2 \in \Omega_1(x)$, thus $y_1, y_2 \phi x$. Let $[y_1, y_2]$(10) denote the segment from y_1 to y_2 and suppose that there exists $y_3 \in [y_1, y_2]$ such that $x \phi y_3$. Since X is convex, $[y_1, y_2] \in X$. Let $\theta_1 = \Omega_1(x) \cap [y_2, y_3]$,

$\Theta_2 = \Omega_2(x) \cap [y_2, y_3]$. Since $[y_2, y_3]$ is connected and Θ_1 and Θ_2 are closed and non - empty, there is a point $y_4 \in \Theta_1 \cap \Theta_2$, so $y_4 \sim x$.

Similarly, there is an other point $y_5 \in [y_2, y_3]$ such that $y_5 \sim x$. But $y_3 \in [y_4, y_5]$ and, since ϕ is convex, $y_3 \, \phi \, y_4 \sim x$, giving a contradiction.

It will be helpful in a further discussion of the gain in generality obtained here, and in gaining further insight in to the above assumptions, to introduce the following argument.

If Δ is a subset of X $(\Delta \subseteq X)$, a point $\delta \in \Delta$ is called *maximal* if $\delta \, \phi \, x$ for all $x \in \Delta$.

LEMMA 3. If Δ is a *closed subset* of X and ϕ is *continuous* then Δ contains a *maximal* element.

PROOF**(11)**. For each $\delta \in \Delta$, let $\Delta_1(x) = \Omega_1(x) \cap \Delta$. The sets $\Delta_1(x)$ are closed and nested by inclusion, hence, by the compactness of Δ there exists $\delta \in \cap_{\delta \in \Delta} \Omega_1(x)$. This is the *desired maximal* element.

We shall discuss some other aspects of the above argument. In the meantime, it will be convenient to introduce and discuss one more bit of terminology before turning to the statement of the main result of this paper. Thus, for the total production set Y, which also lies in R^n, we assume

ASSUMPTION III. Y is a *closed convex* cone.

ASSUMPTION IV. $Y \cap R^n_+ = \{0\}$, where $R^n_+ = \{x \in R^n : x = (x_i)_{i=1,2,\ldots,n} \geq 0\}$ is the *non - negative orthant* in R^n.

The assumption that Y is a cone recognizes the role of constant returns to scale as a basis for pefect competition. On this point see Debreu (1962)**(12)**, and McKenzie (1959)**(13)**. It also may be defended as an approximation when efficient firm sizes are small, and in this sense was accepted by both Marshall and Walras. It may be argued that the error of such an approximation is of the same order as one of those introduced by the assumption of convexity in the presence of indivisible goods. In any case the assumption of convex production sets for firms may be shown to be mathematically equivalent to Assumption III (McKenzie, 1959, pp. 66-67). However, in the Arrow-Debreu formulation, Assumption IV is not a real restriction. In the sense that it amounts to ignoring goods that are available in any desired quantities without cost.

In McKenzie's 1981 theorem, the consumption sets X_i are net**(14)** of initial stocks. There are two aspects of the following assumptions on the relations between the X_i and Y that are of particular interest from the standpoint of the workings of a competitive equilibrium. The first assumption is :

ASSUMPTION V. $X_i \cap Y \neq \varnothing$. Furthermore, there is a common point x_e in the relative interiors of Y and X.

The first part of Assumption V states that any consumer $i \in \{1,2,\ldots,m\}$ can survive without making trade. The second part implies that consumers may choose the price space R^n so that any price p that supports Y will have p . $x < 0$ for some $x \in X$. More precisely, if for instance, p is compatible with equilibrium in a production sector, then there is a feasible trade for the group of all consumers with negative value. This also may be interpreted as saying that some consumer has income, in the sense that he is not on the boundary of his consumption set. We can at this point not that, if we go on assuming rational agents, firms will act to maximize profits and consumers choose to maximize utility. Moreover, the fact that the theory of actual choice is simultaneously a theory of rational choice gives us here one reason to accept this type of interpretaion. An other point that appears eminently reasonable from an economic point of view and to which we shall return later (Section 3).

In the discussion to follow, when we say that $\{L_1, L_2\}$ is a division of the consumers $\{1,2,\ldots,m\}$ into two groups, we shall mean that $\{L_1, L_2\}$ is a partition of $\{1,2,\ldots,m\}$; i.e., that :

$$L_1 \cup L_2 = \{1,2,\ldots,m\},\ L_1, L_2 \subseteq \{1,2,\ldots,m\}, L_1 \neq \varnothing, L_2 \neq \varnothing;\ L_1 \cap L_2 = \varnothing.$$

Then, in the notation of this paper, McKenzie's irreducibility condition can be stated as follows : Suppose there are *m* consumers. Let L_1 and L_2 be nonempty sets of indices for consumers such that $L_1 \cap L_2 = \varnothing$ and $L_1 \cup L_2 = \{1,2,\ldots,m\}$. Let $x_{Lh} = \Sigma_{h \in Lh}(x^i)$ and, for *h* = 1,2. Thus as we mentioned previously, the following single assumption pro-vides at least a slight generalization of the relation between the X_i and Y.

ASSUMPTION VI. However L_1 and L_2 may be selected, if $x_{L1} = y - x_{L2}$ with $x_{Lh} = \Sigma_{h \in Lh}(x^i)$, and $x_{L2} \in X_{L2}$, then there is also $y' \in Y$, and $w \in X_{L2}$, such that $x'_{L1} = y' - x_{L2} - w$ and $x'_i \not\prec x_i$ *or* $x'_i \sim x_i$ for all $i \in L_1$,, and $x'_i \not\prec x_i$ for some $i \in L_1$.

We can now shortly state at this point the standard view of the relations(15) between economics, ethics, and policy. In the sense that economics is linked to both ethics and the theory of rationality, unlike many other sciences. As we will see, although many economists regard economics as a *positive* science of one sort of social phenomenon, economics is also built around a *normative* theory of rationality, and it has a special relevance to policy making. Many major economists, such as Adam Smith in the eighteenth century, John Stuart Mill and Karl Marx in the nineteenth century, and Frank Knight and John Maynard Keynes in the twentieth century, have also been *political* and *moral philosophers*.

This connection is not surprising, since economists are constantly called upon for policy advice, and theoretical commitments in economics are associated with evaluative commitments to forms of social order and to the interests of different social classes. Indeed, it might appear that the *challenge* here is not to demonstrate that ethics and economics are linked but to defend the possibility of introducing the *axiom of rationality* in the general equilibrium walrasian model. Thus we will confine ourselves to the above defintion; that is, *rationality hypothesis* is introduced throughout this paper such as an agent that chooses or acts so as to maximize his or her utility subject to a budget constraint.

Turning now to the above assumption, we first note that the resource relatedness assumption of Arrow and Hahn**(16)** implies this assumption, but the converse is not true. Since they assume that a household can survive with less of all the resources it holds, they are able to take *w* equal to a small fraction of the resources held by L_2 consumers. It is also supposed that L_1 consumers can be benefited with this *w*. As we will see, Arrow and Hahn assume that $X_i \subseteq R^n_+$ for each $i \in \{1,2,\ldots,m\}$. However, the assumption that X_i is bounded below for each *i* is mathematically equivalent to requiring $X_i \subseteq R^n_+$ (although not to the condition $X_i = R^n_+$) ; and hence we're gaining no real generality here. On the other hand, it is frequently convenient to express, e.g., offers of labor services, by negative coordinates; and hence there is some gain in convenience of application by replacing the assumption that $X_i \subseteq R^n_+$ with the assumption that X_i is bounded below for each $i \in \{1,2,\ldots,m\}$. There would also appear to be some generalization effected by many alternative ways of expressing the McKenzie's above irreducibility condition. That is, L_2 consumers may be moved to a preferred position by the addition of a vector *y'* - *y* from the local cone of Y at *y* **plus** a feasible trade from L_2. For details of the argument, see Koopmans (1951, p. 83).

We are now ready to give one of the main results of this paper. It can be stated as follows. Competitive equilibrium is defined by a price vector $p \in R^n$, an output vector *y*, and vectors $(x^i)_{i=1,2,\ldots,m}$ of consumer trades that satisfy

(i) $y \in Y$ and $p \,.\, y = 0$, and for any $y' \in Y$, $p \,.\, y' \leq 0$.

(ii) $x^i \in X_i$ and $p \,.\, x^i \leq 0$, and $x^i \not\prec_i x'$ *or* $x^i \sim x'$ for any $x' \in X_i$ such that $p \,.\, x' \leq 0$, $i \in \{1,2,\ldots,m\}$.

(iii) $\sum_{i \in \{1,2,\ldots,m\}} x^i = y$.

It is obvious that this result generalizes Arrow's and Debreu's 1954 theorem in three principal respects. Condition (*i*) corresponds to Walras' requirement**(17)** in a sense that there should be *"ni bénéfice, ni perte"* in equilibrium. Condition (*ii*) implies that if consumers are rational, they will maximize preference over their budget sets. This is the traditional picture of neoclassical paradigms when defining individual rationality in economics. Condition (*iii*) says that consumer trades sum to the total production. In other words, given p . $y = 0$ and p . $x^i \leq 0$, it follows from the same condition above that p . $x^i = 0$. With these conditions in mind, we can now make the following.

DEFINITION. A competitive equilibrium is a set of vectors (P, y, $(x^i)_{i=1,2,\ldots,m}$) that satisfy the three above conditions.

An economy ξ may then be defined by $\xi = (Y, X_i, \phi_i, i \in \{1,2,\ldots,m\})$. One form of the classical theorem on the existence of a competitive equilibrium as it was proved in the 1950's with the various improvements that have been made since is :

THEOREM. *If an economy* ξ *satisfies the Assumptions* I, II, III, IV, V, *and* VI, *there is a competitive equilibrium for* ξ.

This is the main form of the classical theorem on existence of competitive equilibrium that was promised by our approach. Its major improvements are the removal of the survival assumption based on the work of James Moore and the discard of transitivity of the preference relation based on the work of Sonnenschein, Shafer, Mas-Colell, Gale, and many other writers in this literature. However, note that in *general equilibrium* consumers make choices between entire consumption plans, not between individual commodities. A single commodity has significance to the consumer only in relation to the other commodities he has consumed, or plans to consumer. Together with *transitivity* and *completeness*, this *hypothesis* about consumer preferences embodies the neoclassical ideal of rational choice. Since we are primarily interested in competitive equilibrium under rationality hypotheses, we will avoid the difficulties arised from an extension of the concept in the literature.

Rationality has not always been a primitive hypothesis in neoclassical economics. It was customary(18) to regard satisfaction, or utility, as a measurable primitive; rational choice, when it was thought to occur at all, was the consequence of the maximization of utility. And since utility was often thought to be instantaneously produced, sequential consumer choice on the basis of sequential instantaneous utility maximization was sometimes explicitly discussed as irrational. In the next few pages I shall try to summarize the primitive mathematical concepts, and their economic interpretations, that define the approach we follow. I also give a hint of the arguments used to establish the conclusions to which we will arrive. I think there are advantages to the introduction of these innovations into an exposition that favors intuitive understanding and generalization of the paper results.

3. A WEAKER RATIONALITY CONCEPT.

A great challenge for future general equilibrium models is now how to formulate a sensible notion of *bounded rationality*, without destroying the possibility of drawing our normative above conclusions. Although the rationality principle is in some respects a *weakening* of the hypothesis of measurable utility and instantaneous utility maximization, when coupled with the notion of consumption plan it is also a strengthening of this hypothesis, and a very *strong* assumption indeed. For example, a consumer's preferences do not change according to the role he plays in the process of production, nor do they change depending on an other consumer's preferences, or the supply of commodities. This problem received enormous attention (see Leroux 1993)**(19)** and some claim that it has been solved by weakening the Arrow - Debreu definition of equilibrium to an *"imperfect discriminating power"* concept of the consumers. But this definition is itself suspect; in particular, it may not be implementable.

This brings us to the main question addressed in this article: The rejection of any *non - transitive* preference relation has its roots in the assimilation between the rationality of the agent and the transitivity of his preferences. But, it depends on the nature of the decision that the transitivity of preferences describes the rationality of the agent, as Rawls**(20)**says: *"Our decision is perfectly rational as soon as we face up our context and do our best"*. In particular, the binary choice of the most valuable endowment is certainly rational, even if this criterion doesn't lead to a transitive preference relation. The hostility to a *non - transitive* preference relation is then a consequence of the usual practice in *economics* in general, and in *welfare theory* in particuler. According to the Arrowsian point of view, the social preference is inferred from (or at least compatible with) the individual preferences. But, the point of view adopted here is by no means *consequentialist* but *procedural*.

The key to this distinction is that in procedural theory of rationality, individuals use *rules of thumb* or simple procedures to guide their actions. A point to which we will come back throughout the remainder of the paper.

As we mentioned previously, it is not easy to separate the significance and influence of the Arrow - Debreu model of general equilibrium from that of mathematical economics itself. In an important contribution to general equilibrium theory, Alain Leroux (1993) tries to go further, by maintaining the same approach as Jamison and Lau (1977)**(21)** who studied the consequences of assuming in an exchange economy that consumer preferences are a semiorder (instead of a preorder). He establishes the existence of spots of equilibria (i.e fully dimensioned subsets of equilibria) in exchange economies. Alain Leroux has defined a weaker notion of transitivity which he calls *"pseudo - transitivity"*.

Leroux's strategy for proving existence of equilibrium in this paper is to prove that for some equilibrium (z, p) and in particular for any equilibrium of the standard theory, i.e. with perfect discriminating power, there is a neighborhood $W \times V$ of (z, p) in $R \times \Pi$, such that any $(x, q) \in W \times V$ is an equilibrium with imperfect discriminating power; where W is the set of equilibrium allocations, V the set of equilibrium prices, R the set of feasible allocations, and P the price simplex. The trick is to define a new preference relation that satisfies the usual properties: continuity, monotonicity and convexity. This preference relation is called *"preference relation with perfect discriminating power"* or shortly *D - preferences*, by reference to Debreu (1959). In an other direction, an artificial construction was needed and led him to define a *"preference relation with imperfect discriminating power"*. On this point, see Alain Leroux (1993, p. 432). The greatest triumph of this contribution was to lay out explicitly the conditions under which it is possible to deduce the rationality of the consumer with imperfect discriminating power from the rationality of the ideal consumer with perfect discriminating power. It should be remarked that in a strict mathematical sense the approaches of Leroux and Arrow and Hahn, Moore, and Debreu are equivalent, without resort to approximations, when the last define, by following the same strategy, the closely related notions of a *"compensated equilibrium"*, and a *"quasi - equilibrium"*.

A *quasi - equilibrium* in our setting satisfies the Assumptions (I) and (III) above, but in place of (II) there is :

(II_2) $x_i \in X_i$ and $p \,.\, x_i \leq 0$, and $x_i \not\succsim_i x'$ *or* $x_i \prec_i x'$ for any $x' \in X_i$ such that $p \,.\, x' \leq 0$, or $p \,.\, x_i \leq p \,.\, x'$ for all $x' \in X_i$, $i \in \{1,2,\ldots,m\}$.

A *compensated equilibrium* replaces (II) by :

($II_{2'}$) $x_i \in X_i$ and $p \,.\, x_i \leq 0$, and $p \,.\, x_i \leq p \,.\, x'$ for any $x' \in X_i$ such that $x' \not\succsim_i x_i$ *or* $x' \prec_i x_i$.

While the above argument formally demonstrates the fact that Leroux's approach is a special contribution to the works of Arrow, Hahn and Debreu, it would appear fruitful to notice that the assumption that converts a proof that a quasi - equilibrium exists, given Assumption(II), into a proof of existence for competitive equilibrium is essentially *irreducibility*, that is the Assumption(VI). These assumptions insure that all rational consumers have income at a *quasi - equilibrium*, so the second alternative of(II_2) does not occur and the condition of the same assumption obviously implies(II).

Observe finally, that although a symmetry(22) may exist between the different approaches, there is only one rigorous that characterize the behavior of all the economic agents when entering the market individually.

4. CONCLUSION

At the foundation of both positive and normative economics lies a normative theory of *individual rationality*. The theory seems to be very *thin*, in a sens that it does not raise any questions about the rationality of one's ultimate ends and very few questions about the rationality of beliefs. The standard view of rationality concerns only, as we mentioned previously, the *internal completeness* and *consistency* of an individual's preferences and the connection between preference and choice. Strictly speaking, an agent's preferences are rational in our assumptions only if they are complete and transitive. Though it has been questioned whether transitivity is a requirement of rationality, it is certainly plausible. Amadou's preferences are transitives *if and only if*, for all options α, β, and γ, if Amadou prefers α to β and β to γ, then Amadou prefers α to γ. And Amadou's preferences are complete if for all options a and β, either Amadou prefers α to β or Amadou prefers β to α, or Amadou is indifferent between α and β. Notice that if Amadou's preferences are complete, then Amadou is never unable to rank α and β.

Conversely, if Amadou's preferences are rational, one can assign numbers to the objects of his preferences. These numbers, which are arbitrary apart from their order, merely indicate preference ranking. They are *"ordinal utilities"*, and the theory of rationality may be restated as follows : Amadou is rational *if and only if* his preferences may be represented by ordinal utility functions, and his choices maximize utility. The standard theory of rationality is also silent concerning what to do in circumstances of risk or uncertainty, and indeed neoclassical economics often abstracts from the problems that risk and uncertainty raise. There is a third condition called *"continuity"*, which we shall not discuss here. If, for example, there is an uncountable infinity of options, then completeness and transitivity do not guarantee the existence of a continuous utility function. See Debreu (1959)**(23)**, pp. 54-59.

This paper has explored ways in which *completeness* and *transitivity hypotheses* are relevant to general equilibrium Walrasian model. Many of the connections between the theory and the underlying hypotheses are provided by the paradigm of rationality. These connections demonstrate that there exist other directions in which the theorems of existence could be substantially strengthened. One of them require to remove the transitivity and the completeness of an individual's preferences. Such a process was begun by Sonnenschein (1971) and brought to fruition by Mas-Colell(1974) and Gale and Mas-Colell(1975). Sonnenschein showed that the existence of a well defined demand function does not depend on the transitivity of preference. He also showed that the demand function would be upper semi - continuous if preferences are continuous. Our central conclusions are that rationality is relevant to the agent's behavior when he acts so as to maximize his utility subject to a budget constraint.

Références bibliographiques

Arrow, K. J., and G. Debreu., 1954. "Existence of an Equilibrium for a Competitive Economy", *Econometrica*, 22 , 265-290.

Arrow, K. J., and F. Hahn., 1971.*"General Competitive Analysis"*, San Francisco : Holden Day.

Debreu, G.,1959. *"Theory of Value"*, New York : John Wiley & Sons.

Gale, D., 1955."The Law of Supply and Demand", *Mathematica Scandinavica*, 3 , 155-169.

Gale, D., and A. Mas-Colell., 1975. "An Equilibrium Existence Theorem for a General Model without Ordered Preferences", *Journal of Mathematical Economics*, 2 , 9-15.

Jamison, D. T., and L. J. Lau., "The Nature of Equilibrium with Semi-ordering Preferences, *Econometrica*, 45, 1595-1605.

Kakutani, S., 1941. "A Generalization of Brouwer's Fixed Point Theorem", *Duke Mathematical Journal*, 8, 457-459.

Leroux, A., 1993. "General Equilibrium with Imperfect Discriminating Power", *Journal of Mathematical Economics*, 22 , 431-437.

Mas-Colell, A., 1974."An Equilibrium Existence Theorem without Complete or Transitive Preferences", *Journal of Mathematical Economics*, 1, 237-246.

McKenzie, L. W., 1954."On Equilibrium in Graham's Model of World Trade and Other Competitive Systems", *Econometrica*, 22, 147-161.

McKenzie, L. W., 1981."The Classical Theorem on Existence of Competitive Equilibrium", *Econometrica*, 49, 819-841.

Moore, J., 1975. "The Existence of Compensated Equilibrium and the Structure of the Pareto Efficiency Frontier", *International Economic Review*, 16, 267-300.

Notes

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(1) A. Wald., "Über einige Gleichungssysteme der mathematischen Ökonomie, Zeitschrift für Nationalökonomie 7 (1936), 637-670. Translation to English, Econometrica 19 (1951), 368-403.

(2) K.J. Arrow and G. Debreu., "Existence of an equilibrium for a competitive economy", Econometrica 22 (1954), 265-290.

(3) I.e., for it to have a solution.

(4) I.e., fully dimensioned subsets of equilibria.

(5) A. Mas-Colell., "An Equilibrium Existence Theorem without Complete or transitive Preferences," Journal of Mathematical Economics, 1(1974), 237-246.

(6) H. Uzawa., "Walras' Existence Theorem and Brouwer's Fixed Point Theorem", Economic Studies Quarterly, 13(1962), 59-62.

(7) For details of the argument, see McKenzie (1981), especially part 2 of the proof of the survival assumption, pp. 823-825.

(8) This is the well-known budget inequality which in our notation takes the simple form, $p.U_i(p) \geq 0$. That is, the scalar product of *p* and $U_i(p)$ must be non-negative. Subject to this inequality it is generally assumed that each agent acts so as to maximize his satisfaction.

(9) It is easy to show that this condition is equivalent to an apparently weaker one, namely the existence of neighborhoods V_3 of *x* and V_4 of *x′* such that $x_1 \not\succ x'$ for all $x_1 \in V_3$ and $x \not\succ x_2$ for all $x_2 \in V_4$.

(10) $[y_1, y_2] = \{y : y = \lambda_1 y_1 + \lambda_2 y_2, \lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_1 + \lambda_2 = 1\}$.

(11) The reader will undoubtedly notice that the method of proof developed here owes a great deal to Gale (1955), as well as to Arrow and Debreu (1954).

(12) G. Debreu., "New Concepts and Techniques for Equilibrium Analysis", International Economic Review, 3 (September, 1962), pp. 275-273.

(13) L. W. McKenzie., "On the Existence of General Equilibrium for a Competitive Market", Econometrica, XXVII (January, 1959), pp. 54-71.

(14) That is, the elements of X_i are possible trades.

(15) Economics should be and (apart from individual failing) is in fact a purely *positive* science. Policy making requires both goals, which are influenced by the values of policy makers, and *positive* (*engineering*) *knowledge* of the means to accomplish those goals. Since economics provides this *engineering knowledge*, it is extremely important to policy making, but economics has no connection to ethics whatsoever. So-called *normative* economics is simply the application of positive economics to questions that are of immediate evaluative relevance. Thus the study of *ideology* and of the *values* of economists is irrelevant to understanding economics or economics methodology, though it may help one to understand the scientific failings of particular individuals (Hausman, 1993, pp. 252-277).

(16) K. J. Arrow., and F. Hahn., "General Competitive Analysis", San Francisco : Holden - Day, 1971, p. 117.

(17) L. Walras., "Eléments d'Economie Politique Pure", Paris : Pichon and Durand - Auzias, 1926, p. 225. Translated as Elements of Pure Economics by Jaffé. London : Allen and Unwin, 1954.

(18) On this point, see Bentham, Jevons, Menger, and Walras.

(19) A. Leroux., "General equilibrium with imperfect discriminating power", Journal of Mathematical Economics, 22 (1993), pp. 431-437.

(20) J. Rawls., "A Theory of Justice", Cambridge, Harvard University Press, 1971.

(21) D. T. Jamison and L. J. Lau., "The nature of equilibrium with semi - ordering preferences", Econometrica 45, 1977, pp. 1595-1605.

(22) In the sense that, as Leroux concludes, p. 436 : "The important results, that establish the existence of sets of δ - equilibria (particulary Jamison and Lau), confirm the intuition that local uniqueness of the equilibria vanishes as soon as the consumers are not assumed to discriminate perfectly between different bundles. In other words, inexactness of individual choices make localization of the general equilibrium fuzzier".

(23) G. Debreu., "Theory of value", 1959, Wiley, New York.